*JOURNAL OF DEVELOPMENT ECONOMICS AND FINANCE*
*Vol. 4, No. 1, 2023, pp. 57-78*

*URL : www.arfjournals.com*
*https://DOI:10.47509/JDEF.2023.v04i01.04*

# Testing the Weak Form of Efficient Market Hypothesis in Developing Countries (LDCs) Stock Markets: Limits and Suggestions

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**Article History**

Received : 08 January 2023
Revised : 10 February 2023
Accepted : 02 March 2023
Published : 17 April 2023

**To cite this article**

Kouakou Thiéjé Gaudens-Omer (2023). Testing the Weak Form of Efficient Market Hypothesis in Developing Countries (LDCs) Stock Markets: Limits and Suggestions. *Journal of Development Economics and Finance,* Vol. 4, No. 1, pp. 57-78. https://DOI: 10.47509/JDEF.2023.v04i01.04

***Abstract:*** This paper highlights some limits of weak form of efficiency tests on stock markets in developing countries. These tests, by neglecting important questions such as joined tests and the "bad model problem", don't integrate all the empirical implications of the informational efficiency theory. By using a framework inspiring from Fama (1970) sub-martingale model and the Harrison-Kreps (1981) paradigm, we show the relevance of joined tests using as underlying model an equilibrium model as D-CAPM (downside-Capital Asset Pricing Model) compatible with asymmetry and non-normality of the distribution of returns in developing countries.

***Keywords:*** informational efficiency, efficiency test, joined tests, stock markets, LDCs.

***JEL Classification Numbers:*** G11, G14, G15.

## 1. Introduction

At the heart of the development of financial markets is the important issue of the informational efficiency of stock markets. According to the theory of informational efficiency, the prices of financial securities always reflect, without friction, all available information. Under these conditions, financial assets prices are only due to changes in this information. In other words, a company's stock prices express its fundamental value (dividends, coupons, profits, growth prospects, and so on), which allows for an optimal allocation of capital in the economy, and thus creates wealth in the economy. In addition, the degree of efficiency of the stock markets determines the hedging strategies of investors and fund managers. In fact, the more efficient a market is, the better the investors who intervene in it can hedge via derivatives (Etner and Granger, 2011). It also conditions their investment strategies: in the event of efficiency (or inefficiency), it is impossible (or possible) to beat the market in a systematic and predictable way, which allows passive management (or active management) as the best investment strategy (Jensen, 1978). The efficiency of the stock markets is a real issue for the performance of investment strategies and for the growth of an economy.

For developing countries (LDCs), efficiency is also an economic and social development issue. In fact, portfolio investments should be able to move to those countries with some of the highest return on investment. However, this power of attraction can only be realized if the financial markets are dynamic and efficient (N'Dong, 2007). Thus, the efficient functioning of stock markets in developing countries determines their "investibility". In this context, the evaluation of the efficiency of the stock markets is, beyond the theoretical stake that this represents, of crucial importance for both the participants (national and international investors, private companies, fund managers, etc.) and for the public authorities. This requires the scientific relevance of efficiency tests which, refined over the course of the theoretical debates on efficiency, have resulted in joint tests. However, the efficiency tests conducted in the LDCs consist only of random walk tests and do not incorporate all the empirical implications of the theory of efficiency. They ignore important issues such as joint tests and the problem of the bad model (Fama, 1976). There is therefore a risk of obsolescence of the conclusions of efficiency studies in developing countries.

Our objective, in this article, is to develop an analytical framework that shows the theoretical relevance of joint tests as efficiency tests and the problem of bad model that implies before suggesting efficiency test tracks specific to the LDCs. The rest of the article is organized as follows: Starting from a brief literature review on efficiency, Section 2 shows how the formalization of information efficiency via a sub-martingale model requires the use of joint tests as efficiency tests. In Section 3, inspired by the Harrison-Kreps paradigm (1981), we show that a D-CAPM (Downside-Capital Asset Pricing Model) equilibrium model can serve as a relevant underlying model for leading the tests joined on the stock markets of the DCs, without however failing to underline the problem of bad model. Section 4 concludes the article.

## 2. A literature review on the efficiency of stock markets

### *2.1. From random walk models to sub-martingale models*

The theoretical analysis of efficiency begins with Samuelson (1965) who theoretically reconciles two seemingly contradictory approaches to efficiency. The first, the fundamentalist approach to efficiency, uses the methods of financial valuation of firms and presents stock markets as markets in which the observed price of a financial asset corresponds to the fundamental value of that asset, equal to the discounted value of the dividend flows it reports (Fisher, 1911; William Burr, 1940; Gordon and Shapiro, 1956; Gordon, 1962). Here, observed asset prices provide appropriate signals for the allocation of resources. The second approach to efficiency, that of random walking, assumes that stock prices cannot be predicted from the distribution of past prices. Working (1948) is the first to intuitively compare these two approaches by associating the random walk with the proper functioning of the financial market. The well-priced fair price of a company is therefore the one that makes the market totally unpredictable. In other words, the more the variations of the stock market look like that of the roulette wheel of a casino, the more it means that investors are competent and responsible. Samuelson (1965) mathematically shows that, the more investors correctly evaluate a company using the discounted cash flow model, the more the correctly anticipated prices fluctuate randomly, and the more the stock market variations are "uncorrelated". This fundamental result of Samuelson is often considered, in the financial literature, as the origin of the hypothesis of efficient markets (Fox, 2013).

A third approach to efficiency is based on the microstructure of the financial markets. By giving a microeconomic foundation to the analysis of efficiency, this approach brings very rich and especially intuitive problems of the question of efficiency. It takes into account the real operating conditions of the financial markets. Here, the efficiency of a stock market depends, among other things, on the market structure and the structure of the information. On a competitive stock market characterized by the atomicity of agents and the absence of transaction costs, is ensured infinite liquidity of the market, which can absorb the flow of sales and purchases without changing price dynamics (Biais, 1990). Such perfect market liquidity induces efficiency especially in the case of perfect information which assumes that agents have free access to all available information at no cost. In a situation of information asymmetries between informed agents and uninformed agents, the price of a financial security may reveal private information, thus reflecting the informational efficiency of the market (Grossman, 1976). In the case of imperfect competition, informational price efficiency is reduced when informed agents are aware of their impact on prices (Kyle, 1985, 1989). In particular, the assumption of efficiency of the financial markets may well not hold in the presence of a major swinger, for example a large institutional investor, capable of influencing not only the market price of financial assets but also the beliefs of other swingers (Laffont and Maskin, 1990).

The microstructural approach of the stock markets, emphasizing rigor and coherence to the detriment of empirical operationality, highlights few testable results. This contrasts sharply with the statistical approach of stock markets where empiricism preceded the theoretical formulation. The first tests of efficiency consisted in testing the weak efficiency on the stock markets of developed countries via random walk tests (Working, 1948; Kendall, 1953; Fama, 1965; Cootner, 1965). Indeed, according to the statistical approach, efficient financial markets evolve according to a random walk. Formally, if the price process ($S_t^j$) of the financial asset $j$ at period $t$ follows a random walk, we have: $S_{t-1}^j + \varepsilon_t$. There are several versions of the random walk which are distinguished by the hypotheses relating to the distribution of the errors $\varepsilon_t$ and to which several categories of tests are associated. The first hypothesis assumes independent and identically distributed $\varepsilon_t$ according to a Gaussian distribution $N(0, \sigma_\varepsilon^2)$ (Bachelier, 1900). The random walk in stock market prices was initially viewed as the result of a normal distribution of price changes (Kendall, 1953). Kendall (1953) observes, in an empirical study of the stock market quoted in London between 1928 and 1938, that successive changes in the equity index are completely independent. In the second hypothesis formulated by Fama (1964), the errors $\varepsilon_t$ are independent and identically distributed, but the distribution law of the series is not necessarily a normal distribution. In the latter hypothesis, it is sufficient for the $\varepsilon_t$ to be identically distributed (Granger and Morgerstern, 1963). Debates over the Gaussian or non-Gaussian distribution of stock prices have nothing to do with the hypothesis of a random walk. Indeed, the random walk is compatible with any form of probability law (Fama, 1964). From the outset, the random walk hypothesis was tested and validated on the American market, using serial autocorrelation tests (Cootner 1964; Moore 1964; Fama 1965).

In particular, Fama (1965) shows that the coefficients of autocorrelation between the successive changes of the thirty values of the Dow-Jones index from 1957 to

1962 are very small, whether these changes are weekly, bi-weekly, etc. This hypothesis of independence and the short-term market price trend has also been validated on the British market by Granger and Morgenstern (1963) and Godfrey, Granger and Morgenstern (1964) who use spectral analysis. Other random walk tests include: BDS test, run test, Lo and MacKinlay test, variance ratio test, Chow and Denning test, Lamont test, GARCH-M test, etc. Random walk tests indicate efficiency and not inefficiency. In other words, if the hypothesis of a random walk is not verified, it cannot be concluded that the market is efficient or inefficient. However, the model of the random walk used, based on the Brownian random walk, is too restrictive and does not constitute a true economic model of asset prices. Since it cannot be related to primitive assumptions about preferences and returns, this model cannot be related to optimization-type models that dominate economic analysis (Leroy, 1989). In addition, the Brownian random market model reveals systematic trends in asset prices, thus calling into question certain economic laws, such as the law of the market (Samuelson, 1965). In fact, in this model, asset prices fluctuate around their fundamental value. The prices of the assets, far from equilibrium prices, follow rather a white noise.

In addition, the random walk is not compatible with the stylized fact that in the financial series, the variances of the returns are correlated (Samuelson, 1965, Leroy, 1973). Here, high volatility tends to cause high volatility and small volatilities tend to follow one another. In other words, the prices of securities follow continuous, prolonged periods and turbulent continuous periods of the same size (positive autocorrelation of the successive conditional variances of the prices of the securities but not their successive levels). It is to overcome all these limits that the notion of random walk has been replaced by that of martingale, a less restrictive concept, by resorting to the exponential of Brownian motion (Samuelson, 1965). In addition, the martingale model is a true economic model of asset prices, in the sense that it can be linked to primitive assumptions about preferences and returns (Leroy, 1989). This model of martingale is only valid assuming agents' risk neutrality (Leroy, 1973). But this hypothesis of risk neutrality is necessary only in the exchange economy and no longer in the economics of production where technologies and preferences come into play (Lucas, 1978). Lucas leans more for price processes described by a dividend model than for martingale models. The martingale approach in the strict sense of the term makes it possible to model the price of financial assets by resorting to the notion of fair game (Samuelson, 1965, Fama, 1965).

In a discrete framework, the stochastic process of the asset price $S_t$ is a martingale conditionally to a set of information $\mathcal{F}_t$ if $S_t$ has the following property: for all $t$, $S_t$ is $P$-integrable, and for all $u$, $t$ such that $u < t$, $E_P(S_{t+1} \mid \mathcal{F}_u) = S_t$. Returns follow fair game if and only if the discounted values of future income comprised of the capital gain and dividends (or the anticipated value of the security's price, the expression of the fundamental value) follow a martingale. A stochastic process $R_t$ is a fair game if $E_P(R_{t+1} \mid \mathcal{F}_u) = 0$. Thus, by noting $R_{t+1} = S_{t+1} - S_t$, it follows that the process of the asset price $S_t$ is a martingale conditionally to a set of information $F_t$ if and only if its difference $(S_{t+1} - S_t)$ is a fair game. The notions of martingale and fair game designate a characterization of the equilibrium on the financial markets. In fact, in the martingale model, the price of the asset is assumed to be equal to the fundamental value, unlike the random market model where the price fluctuates around the fundamental value. A fair game concerns the situation of the stock market, which does not believe, at a given moment, the rise or fall of the true price. It is a game with a zero total mathematical expectation: a player cannot be neither advantaged nor injured (Bachelier, 1900). In other words, it is a game where the gain that can be expected to be made at any future time is equal to the sum gained at the present time.

A game where the gain that one can hope to make any time later is greater than or equal to the sum gained at the present moment is defined as a winning game and is modeled by resorting to the sub-martingale model. This sub-martingale model, considered as a model of equilibrium, makes it possible to define the notion of informational efficiency of the financial markets (Fama, 1970). Indeed, when prices follow a sub-martingale, then no transaction rule, even enlightened by information? $\mathcal{F}_t$, can perform better than a simple strategy of buying and selling (Fama, 1970). This reflects the idea that prices fully reflect all available information. Formally, we have: $E_P(S_{t+1} \mid \mathcal{F}_t) \geq S_t$. This result is based on the assumption that the market equilibrium conditions can be specified in terms of expected returns and that the expected returns conditional on the available information are non-negative: $r_{t+1} \equiv \frac{S_{t+1}}{S_t} - 1 \Rightarrow E_P(S_{t+1} \mid \mathcal{F}_t) = [1 + E_P(r_{t+1} \mid \mathcal{F}_t)]S_t$. From this result, it follows that $E_P(S_{t+1} \mid \mathcal{F}_t) \geq S_t$ if and only if $E_P(r_{t+1} \mid \mathcal{F}_t) \geq 0$.

## *2.2. The Need for Joint Tests*

Leroy (1973) shows that considering almost zero autocorrelation tests as in favor of efficiency implies that the yields follow a fair game, which is contrary to its formulation in the form of sub-martingale. Fama (1976), taking note of this devastating criticism of Leroy, shows that autocorrelation tests are compatible with the sub-martingale model of efficiency tests provided that the idea of rational expectations definition is added to the definition of efficiency: a market is efficient if the agents do not neglect any relevant information in the anticipation of the prices of the securities and if the agents form rational expectations (Fama, 1976)[1]. A market is said to be efficient relative to a set of information $\mathcal{F}_t$ if asset returns follow a sub-martingale model and agents form rational expectations. Formally, by noting $r_{t+1}^j$ the profitability of the title $j$ in $t + 1$, the first condition is written: $\varepsilon_{t+1}^j \geq r_{t+1}^j - E_P(r_{t+1}^j \mid \mathcal{F}_t)$. The second condition states that when the market is efficient, all agents form rational expectations with respect to future returns. They correctly anticipate future returns, at equilibrium. As a result, the excess profitability of the asset j has a zero conditional expectation: $E_P(\varepsilon_{t+1}^j \mid \mathcal{F}_t) = 0$ with $\varepsilon_{t+1}^j = r_{t+1}^j - E_P(r_{t+1}^j \mid \mathcal{F}_t)$, the difference between the observed returns $r_{t+1}^j$ and the theoretical returns $E_P(r_{t+1}^j \mid \mathcal{F}_t)$.

In this context, the weak form of efficiency test consists of two steps: on the one hand, testing the independence of the successive returns (here $\mathcal{F}_t$ contains the past values of variables such as asset prices, asset returns); and on the other hand, testing the validity of the forecast equilibrium model which makes it possible to determine the theoretical return $E_P(r_{t+1}^j \mid \mathcal{F}_t)$. As a result, efficiency can be tested only in conjunction with a market equilibrium model. By testing the relevance of the underlying equilibrium model, the joint tests directly test the difference between observed return and theoretical return. In other words, the joint tests directly test whether the returns are correctly anticipated or not. If so, the market is efficient, otherwise it is not efficient. These tests therefore make it possible, unlike the random market tests, to conclude that the financial markets are efficient or inefficient. However, when price changes in the equilibrium model are nil or very low, joint testing is no longer necessary (Fama, 1976, Guerrien and Gun, 2013). This is the case when the time horizon on which the returns relate is the short term and when no new information on fundamentals emerges on the markets. When these two conditions, temporal and informational, are fulfilled, one can neglect to test the relevance of the equilibrium model. As a result, when the efficiency analysis in the short sense is short-term, financial economists are justified in not doing joint tests and restricting themselves to random walk tests, but they are wrong to do so without underlining the temporal and informational conditions of validity of such a test.

However, efficiency studies in developing countries are confined to carrying out random walk tests alone without specifying whether the temporal and informational conditions of such a choice are fulfilled or not. Some of those studies about the stock markets of developing countries conclude to inefficiency. This is the case for the following stock markets: the Casablanca stock market (Bakir, 2002), the Tunisia stock market (Boubaker, 2017), the Ghana Stock Exchange (Ayentimi, Mensah and Naa-Idar, 2013), the Nigeria stock market (Olowe, 1998; Emenike, 2008; Ogege and Udoka, 2012), Jamaica Stock Exchange (May, Rigobert and Tchemeni, 1995), Cameroon, Nigeria, South Africa, Egypt and Kenya stock markets (Forgha, 2012) , the WAEMU[2] Regional Stock Exchange (N'dri, 2007b), etc. These results seem to agree with microstructural observations. For the moment, from a microstructural point of view, stock markets in most developing countries suffer from certain constraints that presume their inefficiency. There are downward trends in liquidity: the amount of securities available for purchase and sale at any given moment is very low. In addition, the lack of depth of the market prevents a real valuation of securities at market prices. This explains why valuations in these markets are among the most expensive in the world, reducing their margin of profitability and ultimately lowering their attractiveness to local and foreign investors. In addition, imperfect competition seems to prevail in these markets, especially as institutional investors monopolize a large percentage of stock market transactions. This high concentration of transactions can also be explained by the low number of IPOs and / or the multiplication of company withdrawals. On the other hand, other efficiency tests conclude to efficiency. These are studies on the Lagos Stock Exchange (Chigozie, 2010), WAEMU (N'dri, 2007a, 2007b, 2015, N'dong, 2007), etc.

Not considering the joint tests may lead to questioning the scientific relevance of the efficiency tests, which would make the conclusions of efficiency studies in the developing countries obsolete. In the next section, we develop an analytical framework that shows the theoretical relevance of the joint tests as efficiency tests and the "bad-model problem" that implies. For this, we use an underlying equilibrium model derived from a martingale model inspired by the paradigm of Harrison and Kreps (1981), and adapted to a context of LDCs. In this model, a consumer-investor maximizes his intertemporal utility by choosing his optimal intertemporal consumption profile, after having evaluated the price of the financial asset in which he invests in a supposedly efficient financial market. To value this financial asset, it takes into account the dividends it allows to report. At the optimum, there are two results: firstly, the discount rate used to discount dividends is an interest rate that does not include a risk premium, that is, the risk free rate; on the other hand, the rational consumer-investor evaluates the price of the asset to its discounted dividend flows. This means that he correctly anticipates the price of the financial asset in which he invests. From there, we try to show how to correctly anticipate the price of a financial asset in an efficient market (low or even zero difference between observed prices and anticipated theoretical prices: $S_t^j = E\left(S_{t+1}^j \xi_{t+1} \middle| \mathcal{F}_t\right)$ makes it possible to deduce that the expected (theoretical) returns of the securities depend on their systematic risk and the return of a reference portfolio.

## 3. An Underlying Theoretical Model Relevant for Developing Countries

### *3.1. The Analytical Framework*

We consider consumer-investors whose utility comes mainly from consumption. At each moment, their wealth is divided into two parts: the first is used for the consumption $c_t^k$ and the second for the investment. Note $e_t^k$, the endowment of the agent k at time t from his salary, for example. For the sake of simplicity, we consider a two-period model with a single financial asset available in terms of investment whose price is noted $S_t$ at time *t*. It is assumed that this asset is exchanged without any constraint. The consumer-investor *k*, endowed with a separable utility function $u(c_t^k) + \delta u(c_{t+1}^k)$, solves the following optimization problem:

$$\begin{cases} \max\limits_{(c_t^k, c_{t+1}^k)} E_t\left[u(c_t^k) + \delta u(c_{t+1}^k)\right] & (1) \\ s.t\, n_t^k S_t = e_t^k - c_t^k & (2) \\ c_{t+1}^k = e_{t+1}^k + n_t^k S_{t+1} & (3) \end{cases}$$

The consumer-investor *k* is supposed to determine his optimal plan of intertemporal consumption by correctly anticipating the price of the financial asset. Specifically, it evaluates the price of the asset to its discounted dividend flows. This method of evaluating the price of the financial asset is formalized by resorting to the notion of valuation core $\xi_{t+1}$ which can be considered as a discounting factor at a random rate[3]. The price of the asset j en t is written:

$$S_t^j = E\left(S_{t+1}^j \xi_{t+1} \,\middle|\, \mathcal{F}_t\right), \quad \xi_{t+1} > 0 \tag{4}$$

This expression of the discounted value of the asset price is that of a martingale that corresponds to the condition of no arbitrage opportunity. This condition, which is nothing other than the first fundamental theorem of finance, ensures that the theoretical prices determined via the discounted cash flow method are in fact equilibrium prices. In addition, all of these assumptions suggest that the financial market in this model is efficient in the fundamentalist sense of the term.

The utility function of the consumer-investor is easily generalized in the case of an infinite time horizon. The first constraint means that the initial endowment is divided entirely between consumption and investment. Since the model has two periods, in the next moment the investor consumes all his wealth from his investment and his salary. The first order condition gives:

$$S_t u'\left(c_t^k\right) = E_t\left[\delta u'\left(c_{t+1}^k\right) S_{t+1}\right] \tag{5}$$

This equation is interpreted as follows: at equilibrium, the loss of utility at the period t caused by the holding of an additional asset should be equal to the utility gain expected at the moment $t + 1$ discounted by $\beta$ to have comparable terms. This equation is true in a general framework and does not depend on the formulation of the model described above. Indeed, the loss of utility at time $t$ due to an additional holding h of assets with very small h is given by:

$$u\left(c_t^k - hS_t\right) - u\left(c_t^k\right) = -hu'\left(c_{t+1}^k\right) S_{t+1} + o(h) \tag{6}$$

On the other hand, holding this additional amount of assets will increase the expected utility at the instant $t+1$ of:

$$E_t\left[u\left(c_{t+1}^k\right) + hS_{t+1} - u\left(c_{t+1}^k\right)\right] = hE_t\left[u'\left(c_{t+1}^k\right) S_{t+1}\right] + o(h) \tag{7}$$

In a market where the agents are rational and where h can be as small as one wants, the sum of utility lost with that which one hopes to gain after discounting must be equal. This restores the equation already obtained (…) by noting that$o(h) \rightarrow 0$, if $h \rightarrow 0$. The result is therefore independent of the time horizon assumed here or the endowments of the introduced agents. For empirical studies, aggregated variables are generally used. The simplest assumption is to assume that all the agents are identical so that the index $k$ can be suppressed. In this case, the value of the price $S_t$ is given by:

$$S_t = E_t\left[\frac{\delta u'(c_{t+1})}{u'(c_t)} S_{t+1}\right] \tag{8}$$

$c_t$ represents the consumption of the representative agent. The hypothesis of homogeneity of agents may at first seem restrictive. In certain situations, an equilibrium model with heterogeneity can be reduced to a framework of homogeneity of the agents in the sense that equilibrium prices and aggregate consumption remain the same in both types of model. It remains to specify a form of utility for the representative agent. Often the function is chosen in the class HARA (Hyperbolic Absolute Risk Aversion) which has the property of being stable in terms of aggregation. A very special case used is the power utility function given by:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \tag{9}$$

Where γ represent the risk aversion coefficient of the agent. In that case, the price $S_t$ is given by:

$$S_t = E_t\left[\delta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} S_{t+1}\right] \tag{10}$$

$$S_t^j = E\left(S_{t+1}^j \frac{\delta u'(c_{t+1})}{u'(c_t)} \mid \mathcal{F}_t\right) = E\left(S_{t+1}^j \delta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \mid \mathcal{F}_t\right) \tag{11}$$

The evolution of the asset price $S_t$ according to the equation above is that which optimizes the usefulness of the representative consumer. Taking into account the no arbitrage condition: $S_t^j = E\left(S_{t+1}^j \xi_{t+1} \mid \mathcal{F}_t\right), \quad \xi_{t+1} > 0$, we derive an expression from the evaluation core:

$$\xi_{t+1} = \frac{\delta u'(c_{t+1})}{u'(c_t)} = \delta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} \tag{12}$$

The evaluation core at the risk-free rate is similar to the Radon-Nikodym operator[4] that allows the transition from the historical probability *P* to the neutral risk probability *Q* (universe of psychological neutrality with respect to risk) by means of stochastic integrodifferential calculus[5]. In this dual world of probability *Q*, a psychological universe in which individuals are indifferent to both the chance of gain and the risk of loss, discounting at an interest rate does not include a risk premium. The evaluation core incorporates in its expression agent preferences and economic factors.

**Proposition 1:** At the optimum of the consumer-investor, the optimal intertemporal consumption profile he chooses, in the absence of arbitrage opportunity, is compatible with a price of the financial asset (in which he invests) valued at a discount rate equal to the risk-free rate, i.e an interest rate that does not include a risk premium. In addition, the no arbitrage condition also shows that the rational consumer-investor chooses the optimal intertemporal consumption profile by valuing the asset price at its discounted dividend flows. This means that it correctly anticipates the price of the financial asset in which it invests: the financial market is efficient.

From there, we try to show how the fact of correctly anticipating the price of a financial asset in an efficient market makes it possible to deduce that the expected (theoretical) returns of the securities depend on their systematic risk and on the return of a reference portfolio. Now, rather than working on the price of securities, we use, for convenience, returns to securities, which have the characteristic of not having a tendency and being close to a white noise. Assuming that the dividends are included in the asset *j* pay-off $S_{t+1}^{j}$, the return $r_j(t+1)$ of this asset is written:

$$r_j(t+1) = \frac{S_{t+1}^{j} - S_t^{j}}{S_t^{j}} \tag{13}$$

We show that at the equilibrium of the consumer-investor, the evaluation core $\xi_{t+1}$ and the asset *j* return are constrained by their product under the mathematical expectation which must be worth 1, namely[6] :

$$E_t\left(\left(1 + r_{t+1}^{j}\right)\xi_{t+1}\right) \triangleq E_P\left(\left(1 + r_{t+1}^{j}\right)\xi_{t+1} \,\middle|\, \mathcal{F}_t\right) = 1 \tag{14}$$

Let $r_t$ be the deterministic profitability of the risk-free asset[7] over the period $[t, t+1]$; remembering that the risk-free evaluation core is a discount factor at a random rate, it comes:

$$E_t(\xi_{t+1}) = \frac{1}{1 + r_t} \tag{15}$$

In this case, equation (1) makes it possible to obtain the expression of the expected excess return of the asset *j*:

$$E_t\left(r_j(t+1)\right) - r_t = -cov\left(r_j(t+1), \xi_{t+1}(1 + r_t) \,\middle|\, \mathcal{F}_t\right) \tag{16}$$

Consider an asset that replicates at the present time the discounted income streams generated by the risk-free asset $\xi_{t+1}.(1+r_t)$. Noting $L_{t+1}$ this replicable asset, by virtue of the second fundamental theorem of finance, it comes: $L_{t+1} = \xi_{t+1}.(1 + r_t) \Rightarrow E_t(L_{t+1}) = E_t(\xi_{t+1}.(1 + r_t)) = 1$. We then obtain:

$$E_t\left(r_j(t+1)\right) - r_t = -cov\left(r_j(t+1), L_{t+1} \mid \mathcal{F}_t\right) \tag{17}$$

Noting $R_{L_{t+1}}$ the return of this replicable portfolio on $[t, t+1]$, it comes:

$$E_t\left(r_j(t+1) - r_t\right) = -cov\left(r_j(t+1), L_t(1 + R_{L_{t+1}}) \mid \mathcal{F}_t\right) = -L_t cov\left(r_j(t+1), R_{L_{t+1}} \mid \mathcal{F}_t\right) \tag{18}$$

Empirical studies have shown that in emerging markets and developing countries, the distribution of returns is very often not symmetrical and that of a normal distribution. In this case, an appropriate measure of risk lies not in the variance but in the semi-variance (Estrada, 2002; Ndong, 2007). The semi-variance distinguishes the up and down phases and is better suited to appreciate the downward and upward movements, sometimes sudden, of the financial markets of developing countries. The variance in a bear market is, according to Sortino and van der Meer (1991):

$$cov\left(R_{L_{t+1}}, R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right) = Var\left(R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right) \tag{19}$$

Where $u_m$ is the market average return. The covariance on such a market is noted: $cov\left(r_j(t+1), R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right)$ such that the equation (11) becomes:

$$E_t\left(r_j(t+1) - r_t\right) = -L_t cov\left(r_j(t+1), R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right) \tag{20}$$

Consider that the replicable asset $L$ in question is the reference asset in the financial market. The excess profitability of this reference asset is determined by putting in the above equation that $r_j(t+1) = R_{L_{t+1}}$. We obtain:

$$E_t\left(R_{L_{t+1}} - r_t\right) = -L_t cov\left(R_{L_{t+1}}, R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right) = -L_t Var\left(R_{L_{t+1}} \quad R_{L_{t+1}} < u_m, \mathcal{F}_t\right) \tag{21}$$

We have:

$$-L_t = \frac{E_t\left(R_{L_{t+1}} - r_t\right)}{Var\left(R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right)} \tag{22}$$

Finally, equation (3) can be written:

$$E_t\left(r_j(t+1) - r_t\right) = \frac{cov\left(r_j(t+1), R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right)}{Var\left(R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t\right)}\left(E_t\left(R_{L_{t+1}} - r_t\right)\right) \tag{23}$$

This relationship shows that the excess of conditional profitability of any asset j relative to the risk-free rate (risk premium) is proportional to that of the reference portfolio. In other words, there is a relationship of proportionality between the risk premiums of each risky asset and the risk premium of the valuation core. The proportionality factor $\frac{cov(r_j(t+1), R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t)}{Var(R_{L_{t+1}} \mid R_{L_{t+1}} < u_m, \mathcal{F}_t)}$ measures the correlation between the title *j* and the reference portfolio L. It corresponds to the beta, in a bear market, denoted $\beta_t^{j,L(-)}$, a measure of systematic risk in such a market. We define the coefficient $\beta_t^{j,L(-)}$, systematic risk in a bear market or downside risk, as the risk of losing more than the market. Thus, equation (23) can be reduced to:

$$E_t\big(r_j(t+1) - r_t\big) = \beta_t^{j,L(-)} E_t\big(R_{L_{t+1}} - r_t\big) \quad (24)$$

This is the D-CAPM (downside-Capital Asset Pricing Model) equation (Estrada, 2002a). It is a suitable variant of Sharpe's (1964) CAPM model (Sharpe Asset Value Model). The higher $\beta_t^{j,L(-)}$, the greater the risk of loss relative to the market, and the higher the risk premium over the conventional CAPM premium.

This equation (23) is a theoretical model of equilibrium making it possible to predict the profitability of the asset. It gives the (theoretical) expected returns of the securities as depending on their systematic risk and the performance of a reference portfolio.

The expression (24) shows that, in case of financial market efficiency, the existence of a small or no difference between observed prices and expected theoretical prices (correct anticipation of the price of financial assets by the consumer-investor: $S_t^j = E\big(S_{t+1}^j \xi_{t+1} \mid \mathcal{F}_t\big)$ is compatible with the result from the underlying equilibrium model according to which the expected (theoretical) returns of the securities depend on their systematic risk and the performance of a reference portfolio. This implies that testing the efficiency of the financial markets amounts to testing both the differences between the theoretical and the observed prices and the underlying equilibrium model. Efficiency is achieved if the two following hypotheses are simultaneously satisfied: 1. the difference between theoretical prices and observed prices is not significant in the statistical sense; 2. the equilibrium model is relevant. In other words, the efficiency tests relate to two hypotheses, one on the underlying model, the other on the deviations: we speak of test on the joint hypotheses or joint test.

In a bull market, systematic risk or upside risk is the risk of gaining more than the market. It is qualified opportunity gain and is equal to:

$$\beta_t^{j,L(+)} = \frac{cov\left(r_j(t+1), R_{L_{t+1}} \middle| R_{L_{t+1}} > u_m, \mathcal{F}_t\right)}{Var\left(R_{L_{t+1}} \middle| R_{L_{t+1}} > u_m, \mathcal{F}_t\right)} \qquad (25)$$

The equation of the U-CAPM model (upside-Capital Asset Pricing Model) is:

$$E_t\left(r_j(t+1) - r_t\right) = \beta_t^{j,L(+)} E_t\left(R_{L_{t+1}} - r_t\right) \qquad (26)$$

The higher $\beta_t^{j,L(+)}$, the greater the gain and the lower the risk premium compared to the classic CAPM premium.

**Proposition 2:** Anticipating correctly the price of a financial asset in an efficient market (low or even zero difference between observed prices and anticipated theoretical prices: $S_t^j = E\left(S_{t+1}^j \xi_{t+1} \middle| \mathcal{F}_t\right)$ makes it possible to deduce that the expected (theoretical) returns of the securities depend on their systematic risk and on the performance of a reference portfolio. Thus, the efficiency of the financial market implies the relevance of the joint tests as a underlying model, a D-CAPM equilibrium model (downside-Capital Asset Pricing Model) compatible with the asymmetry and non-normality of the distribution of returns in developing countries.

### *3.2. The bad-model problem*

We have seen that the joint tests, although necessary to carry out the efficiency tests, can be limited by the bad model problem. The bad model problem appears when the equation (24) (or (26)) is not satisfied. This can come from the wrong specification of the evaluation core because of a bad choice of the utility function for example. In this case, the power utility function is not convenient for modeling agent preferences. Other HARA functions can be considered to see if the problem will persist. If it persists, one sees again the very approach of additively separable utility which may not be suitable. Consumption at time t may have an influence on the marginal utility at $t+1$. In this case, it is better to take more general forms of non-separable utility functions.

If the evaluation core is specified, the problem may be at the level of the data used and not at the agent preference level. One could also explain the non-satisfaction of the equation (24) (or (26)) by the inefficiency of the market due to an irrationality of the agents for reasons of behavioral biases or because the coefficient of risk aversion $\gamma$ has unreasonable values. But this explanation is generally rejected, especially by the proponents of efficiency. In general, when the results obtained are not consistent with the equation (24) (or (26)), Fama and French (op.cit.) retain the poor quality of the underlying long-run equilibrium model as an explanation. Indeed, according to them, when the period of analysis gets longer, the theoretical model of underlying equilibrium becomes more and more bad (Fama, 1998). In other words, the problem of the bad model, less serious when the analysis concerns short-term returns (a few days), worsens when the horizon widens: "A problem of bad model that generates an abnormal average abnormal return $x$% per month eventually becomes statistically significant when the placement is made over several months" (Fama, 1998).

If the bad model problem still persists, these authors recommend adding other macroeconomic factors or indices that can affect the marginal utility of the agents, thus moving from an underlying model with single one factor to a underlying model with several factors. Fama (1992) states that the one-factor model of the CAPM no longer served as an underlying equilibrium model for the attached tests because of its poor long-term quality [8]. Hence the need to replace it with a multi-factor model. An example of a multi-factor model is the APT model (Arbitrage Price Theory) developed by Ross (1976); but Fama and French (1992) do not retain this model which, admitting arbitration, is not a model of equilibrium. They develop a three-factor model: the beta, the Price Earning Ratio (PER) and the Book-to-Market [9]. A little later, Fama and French (2004) develop a five-factor model, confirming, according to them, the poor quality of the CAPM.

In summary, the analysis of the bad model problem concludes that the joint tests could thus dispense with the CAPM which would correspond to an incorrect formulation of the underlying equilibrium model when the time horizon is lengthening. While many dispute this finding that the CAPM is misquoting the equilibrium model in developed countries, this does not appear to be the case in developing countries (LDCs) where several studies show that the CAPM (or CAPM) does not work. Thus, if we follow the reasoning conducted so far, it comes that the efficiency tests on the stock markets in developing countries should resort to joint tests that are based on valuation models assets specific to the LDCs as a relevant equilibrium model. This type of models exists. For example, the D-CAPM (or downside-Capital Asset Pricing Model) model uses semi-variance, not variance, as an appropriate measure of risk in developing countries where the distribution of returns is very often not symmetrical and that of a normal distribution (Estrada, 2002a). Other equilibrium models that capture an asymmetric risk measure include the Lower Partial Moment-CAPM (LPM-CAPM) and the Assymetric Response Model-CAPM (ARM-CAPM). These or other models can help to minimize the bad model problem and to test the efficiency more appropriately on the stock markets of developing countries.

## 4. Concluding Remarks

This paper has shown that the weak form efficiency tests in the stock markets of LDCs are too hasty to achieve efficiency or inefficiency, without highlighting the difficulties associated with this type of test. A limitation of this type of tests consisting mainly of tests of the random walk lies in the following fact: if the independence of the returns implies a very small difference between the observed return and the anticipated return, the dependence of the returns does not necessarily imply that this gap is not small. In other words, when return does not follow a random walk, nothing is known about the difference between observed return and expected return. Thus, the random walk tests only allow to conclude at low efficiency when the random walk is validated. But if the random walk is not validated, it does not mean that this market is not efficient. In fact, we do not know. Random walk tests indicate that financial markets are efficient, but not inefficient. A second limitation lies in the problem of the joint hypothesis and the bad model problem. Indeed, if the joint tests are not necessary when the time horizon on which the stock market returns is the short term and when no new information on fundamentals emerges on the markets, they become long term. There, the price variations of the equilibrium model are no longer null or very weak. The bad model problem can be minimized in developing countries by taking as a relevant equilibrium model, asset evaluation models specific to LDCs: D-CAPM, LPM-CAPM, ARM-CAPM, and so on.

However, taking into account relevant joint tests does not definitively solve the difficulties related to efficiency tests. Findings from efficiency studies using random walk tests are sensitive to small changes in the data structure. Specifically, the results for efficiency (resp. inefficiency) can be easily reversed to confirm inefficiency (resp. efficiency), with simple changes in data frequency, interval studies and data translations for a fixed frequency (Konté, 2010). As a result, it becomes difficult to conclude for efficiency or inefficiency. Debates in this field often derive from ideological positions between proponents of efficiency and those of market inefficiency. Proponents of efficiency invoke the bad model problem to avoid the conclusion of inefficiency when the joint hypothesis is not validated. Proponents of inefficiency admit the relevance of the underlying equilibrium model to the conclusion of inefficiency in case of non-validation of the joint hypothesis. It is in this intellectual quarrel that the 2013 Nobel Prize for Economics was awarded simultaneously to Fama (leader of the supporters of efficiency) and Shiller (leader of the proponents of inefficiency), without the Nobel Committee mentions their work on the informational efficiency of the financial markets as one of the criteria of choice (Guerrien and Gun, 2013). This denotes the difficulty of deciding between the two positions.

And yet stock markets in developing countries suffer from certain constraints that presume their inefficiency. The awareness of this Achilles heel of the stock markets of developing countries invites a greater nuance in the tests of low efficiency carried out on these markets. In this context, it may be useful to test the existence or otherwise of regularities on the stock markets. Indeed, the non-rationality of investors, the overconfidence of the operators, the asymmetry of information, etc., create regularities in the evolution of prices. However, the presence of regularities in the data in high frequency leads to refute the efficient markets hypothesis. Under the hypothesis of efficiency, it would be impossible to find such regularities, impossible to predict the evolution of the markets and impossible to beat a passive trading strategy (buy and hold). These regularities are statistically detectable and can be analyzed via wavelet theory. We reserve this analysis for future research.

## *Notes*

1. Leroy (1973) shows that the equation of the evolution of the return of the security according to its expected value, used by Fama (1970), is nothing more than a tautology. It stems from the way the rate of return is defined! Recognizing these limitations, Fama (1976: 143) uses a theoretical model of the underlying equilibrium as a prediction model.
2. West African Economic and Monetary Union regrouping eight countries: Ivory Coast, Senegal, Mali, Burkina Faso, Niger, Togo, Benin, Guinea.
3. This evaluation core at the risk-free rate is another expression of the Arrow and Debreu elementary securities prices.
4. This operator is an operator of probability change noted $L$ defined in statics by $L = Q/P$, $P$ being the historical probability.
5. The possibility of transforming martingales written under probability P with a discount rate including a risk premium in martingales with probability Q and a discount rate without risk premium is the mathematical trace of the existence of a arbitrated market at equilibrium, that is to say in which there is no longer any arbitrage to be carried out (no arbitrage opportunity).

6. This constraint makes it possible to define the ways of testing, both profitability levels and volatility levels (Hansen and Jagannathan inequality) of the prices on the financial markets.
7. The assumption of a risk-free asset can be lifted; it is sufficient to consider an asset that is not correlated to the evaluation core (active in the zero beta portfolio).
8. According to Eric Berg of the New York Times, Fama alleged that "the beta as sole variable explaining the stock market returns is dead" (Fisher).
9. The PER is the ratio of the price on the net profit per share and measures if the action is expensive or inexpensive: how much the action is worth the profit? Book-to-market, the ratio between the share price and the share value recorded in the books, seeks to know if the stock underestimates or not the value of the company. In the conventional financial theory, these two ratios should not exist, only the beta should count. If they appear, which can lead to profit taking, they would disappear immediately because of the efficiency of the market.
10. That is, the segmentation of the study sample into two equal parts.

## References

Ayentimi, D.T., Mensah A. E., Naa-Idar, F. (2013). « Stock market efficiency of Ghana Stock Exchange: an objective analysis", *International Journal of Management, Economics and Social Sciences*, vol. 2(2), pp. 54-75.

Bachelier, L. (1900). « Théorie de la spéculation », *Annales de l'Ecole Normale Supérieure*, 17, p. 21-86.

Bajeux-Besnaino, I. and R. Portait (1997). « The numeraire portfolio: A new perspective on financial theory", *The European Journal of Finance* 3: 291-309, 1997.

Bakir, K. (2002). « L'efficience des marchés boursiers des pays émergents : l'exemple de la bourse de Casablanca », *Working paper.*

Biais, B. (1990). " Formation des prix sur les marchés de contrepartie », *Revue économique*, 5, p. 755-788.

Black, F. (1993). « Beta and Return: Announcements of the death of beta seem premature », *Working Paper*, pp.74-84.

Boubaker, A. (2017). « Tests de l'efficience informationnelle sur le marché boursier tunisien », *Working Paper.*

Campbell, J. Y., Shiller, R. J. (1988a). "Stock Prices, Earnings, and Expected Dividends," *Journal of Finance*, 43, 661-676.

Chigozie, O.G. (2010). « Analysis of weak-form efficiency on the Nigerian stock market: further evidence from GARCH model », *The International Journal of Applied Economics and Finance*, 4: 62-66.

Cochrane, J. H. (2005). "*Asset Pricing (Revised)*", Princeton University Press, 568 p.

Cootner, P.H. (1964). *"The Random Character of Stock Market Prices"*. Cambridge: MIT Press.

Emenike, K.O. (2008). "Efficiency across time: evidence from the Nigerian stock exchange", *Working Paper*.

Estrada, J. (2002a). "The cost of equity in emerging markets: a downside risk approach*",* *Emerging Markets Quarterly*, Fall, pp. 19-30.

Estrada, J. (2002b). "Systematic risk in emerging markets: the D-CAPM*", Emerging Markets Review*, vol. 3(4), pp. 365-379.

Etner, F., Granger, T. (2011). *"Economie financière"*, Economica, 572 p.

Fama, E.F. (1965). "The Behavior of Stock Market Prices", *Journal of Business*, XXXVIII,34-105.

Fama, E. F. (1965). "Random walks in stock market prices", *Financial Analysts Journal*, 21:55-60,

Fama, E.F. (1970). "Efficient capital markets: a review of theory and empirical work", *Journal of Finance*, 25, pp. 383-417.

Fama, E.F. (1976). *"The Foundations of Finance"*, Basic Books, New York.

Fama, E.F. (1998). "Market efficiency, long-term returns, and behavioral finance", *Journal of Financial Economics* 49, 283—306.

Fama, E.F. et French, K (1988). « Permanent and temporary components of stock prices », *Journal of Political Economy*, 96, 246-27.

Fama, E.F. and French, K.R. (1992). "The cross-section of expected stock returns", *Journal of Finance*, 47, pp. 427-465.

Fisher, I. (1911). *"The Purchasing Power of Money, Its Determination and Relation to Credit, Interest and Crises",* New York: Macmillan.

Forgha, N.G. (2012). "An investigation into the volatility and stock returns efficiency in African stock exchange markets", *International Review of business Research Papers*, vol. 8, n°5, pp. 176-190.

Fox, J. (2013). *"Le mythe du marché efficient"*, Valor, 396 p.

Gordon, M.J. (1962). « The Savings, Investment, and Valuation of a Corporation", *Review of Economics and Statistics*, 44, 37-51.

Gordon, M.J., Shapiro, E. (1956). « Capital Equipment Analysis: The Required Rate of Profit", *Management Science*, vol. 3, issue 1, 102-110.

Granger, C.W.J, Morgerstern, O. (1963). "Spectral Analysis of New York Stock Market Prices", *Kyklos*, XVI,1-27.

Grossman, S. (1976). "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information", *The Journal of Finance,* Vol. 31, No. 2, pp. 573-585.

Guerrien, B. and O. Gun (2011). "Efficient Market Hypothesis: what are we talking about?" *Real World Economics Review* n°56.

Guerrien, B. et Ozgur Gun (2013). « L'étrange silence du *Nobel Prize Committee* sur la « théorie des marchés efficients » », *Revue de la régulation*, 14, 2e semestre, Autumn.

Harrison,J.M., Kreps, D.M. (1979). "Martingales and arbitrage in multiperiod securities markets", *Journal of Economic Theory*, vol. 20, 381-408.

Harrison,J.M., Pliska, S. (1981). "Martingales and stochastic integrals in the theory of continuous trading", *Stochastic Processes and their Applications*, 11, 215-60.

Jensen, M.C. (1978). "Some anomalous evidence regarding market efficiency", *Journal of Financial Economics*, vol. 6(2-3), June-September, pp. 95-101.

Jouini, E. and C. Napp (2007). "Consensus consumer and intertemporal asset pricing with heterogeneous beliefs", *The Review of Economic Studies*, 74(4) pp. 1149-1174.

Kendall, M, G. (1953). "The Analysis of Economic Time Series," *Journal of the Royal Statistical Society* (Series A), XCVI,11-25.

Konté, M. (2010). *"Investisseurs et Marchés Financiers : du comportement des agents à la formation de prix d'équilibre »*, Thèse de doctorat Université Panthéon-Sorbonne, 147 p.

Kyle, A.S. (1985). « Continuous Auctions and Insider Trading", *Econometrica,* vol. 53, No. 6, Nov., pp. 1315-1335.

Kyle, A.S. (1989). « Informed speculation with imperfect competition", *Review of Economic Studies*, vol. 56, n°3, July, pp. 317-355.

Laffont, J.J. and Maskin, E. (1990). "The Efficient Market Hypothesis and Insider Trading on the Stock Market," *Journal of Political Economy*, 98, 70-93.

Leroy, S.F. (1973). "Risk aversion and the martingale property of stock prices", *International Economic Review*, 14(2), pp. 436-446.

Lucas, R. (1978). "Asset prices in an exchange economy", *Econometrica*, 46(6), november, pp. 1429-1445.

Mai, H.M., Rigobert, M.J., Tchemeni, E. (1995), « Prévisibilité des rentabilités sur le marché jamaïcain des actions »,

Moore, A. (1962). *"A Statistical Analysis of Common Stock Prices"*, unpublished PhD dissertation, Graduate School of Business, University of Chicago.

Ndong, B. (2007). *"Marchés boursiers émergents et problématique de l'efficience : le cas de la Bourse Régionale des Valeurs Mobilières (BRVM)"*, Thèse de doctorat de l'université de Franche Comté, 394 p.

N'Dri, K.L. (2015). "Variance Ratio Tests of the Random Walk in the BRVM", *Applied Economics and Finance* Vol. 2, No. 2, May, p. 118-125

N'Dri, K.L. (2007a). "Prévisibilité des rentabilités boursières: cas de la BRVM" *African Review of Money Finance and Banking,* (2007), pp. 39-54.

N'Dri, K.L. (2007b). "Stock Market returns and volatility in the BRVM", *African Journal of Business Management*, 1 (5): 107-112.

Ogege, S., Udoka, C.O. (2012). "The efficiency of the Nigerian Stock Exchange: a theoretical and empirical Analysis", *Indian Journal of Finance*, vol. 6 (9).

Olowe, A.R. (1998). "Stock splits and the efficiency of the Nigerian stock market", *African Review of Money, Finance and Banking*, n°1/2, pp. 97-126.

Poterba, J., Summers, L. (1988). « Mean reversion in stock prices: evidence and implications" *Journal of Financial Economics*, vol. 22 (1), p. 27-59.

Ross, S.A. (1976). "The arbitrage theory of capital asset pricing", *Journal of Economic Theory*, vol. 13(3), December, pp. 341-360.

Samuelson, P. A. (1965). "Proof that properly anticipated prices fluctuate randomly", *Industrial Management Review*, 6: 41-49.

Samuelson, P. A. (1973). "Proof that properly discounted present values of assets vibrate randomly", *Bell Journal of Economics*, 4: 369-374.

Sharpe, W.F. (1964). « Capital asset prices: a theory of market equilibrium under conditions of risk", *Journal of Finance*, vol.19 (3): 425-442.

Shiller, R.J. (1981). « Do Stock Prices Move Too Much to be Justified by Subsequent Changing in Dividends? », *American Economic Review*, Vol. 71, June.

Sortino, F., van der Meer, R. (1991). "Downside Risk", *Journal of Portfolio Management*, 17(4), pp. 27-31.

Stachowiak, C. (2004). « Prévisibilité des rentabilités boursières. Une étude empirique du marché boursier français sur données intraquotidiennes », *Économie & prévision*, n°166, 2004-5. pp. 71-85.

Thomas, A. (1991). « Estimation d'un modèle CAPM avec primes de risque variables dans le cas de la France », *Annales d'économie et de statistique*, n°22, pp. 153-163.

Williams J.B. (1940). *"The Theory of Investment Value"*, Harvard University Press.